Weyl trees of famous irrational numbers

by

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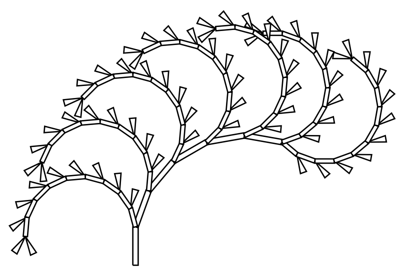
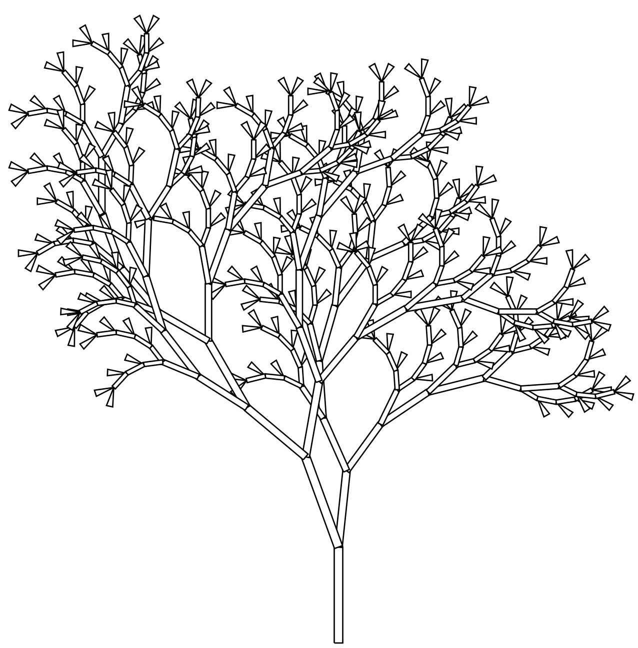
These trees were created by inserting the sequence $\{\theta\}, \{2\theta\}, \{3\theta\}, \ldots$ into a binary search tree, where θ is an irrational number and $\{x\}$ denotes the fractional part of x. For example, $\{\pi\} = \{0.14159\ldots\}$. External nodes are drawn as small isoceles triangles and internal nodes are drawn as rectangles. The size and angle of a given rectangle are proportional to the Horton-Strahler number of the corresponding node.

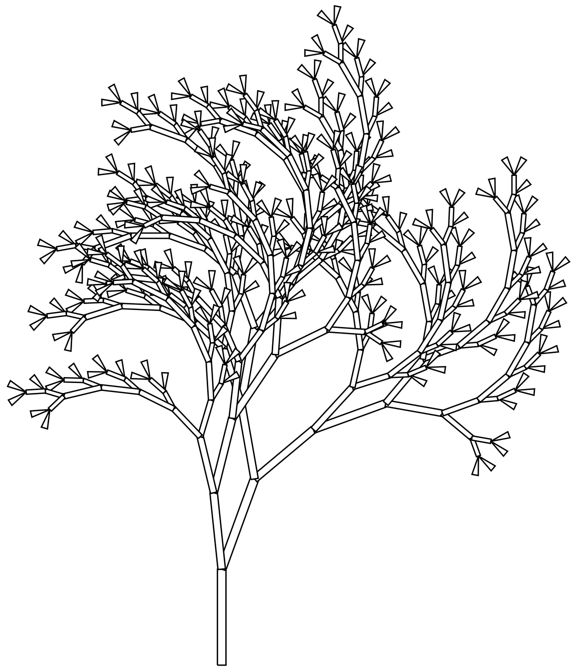
If θ is irrational, the sequence $(n\theta)$ is not periodic. However, due to PostScript's floating-point representation of real numbers, this is not true in our program. We have kept the number of nodes small enough so that this does not affect the drawings.



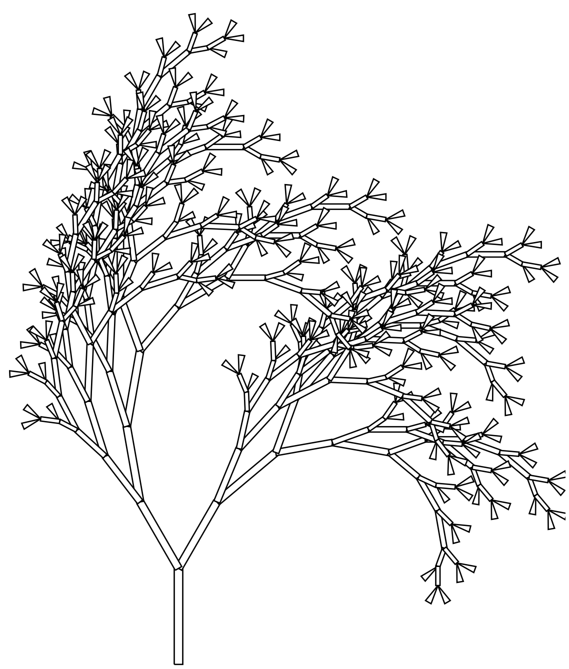
$\pi = 3.14159\,26536-$ (70 nodes)



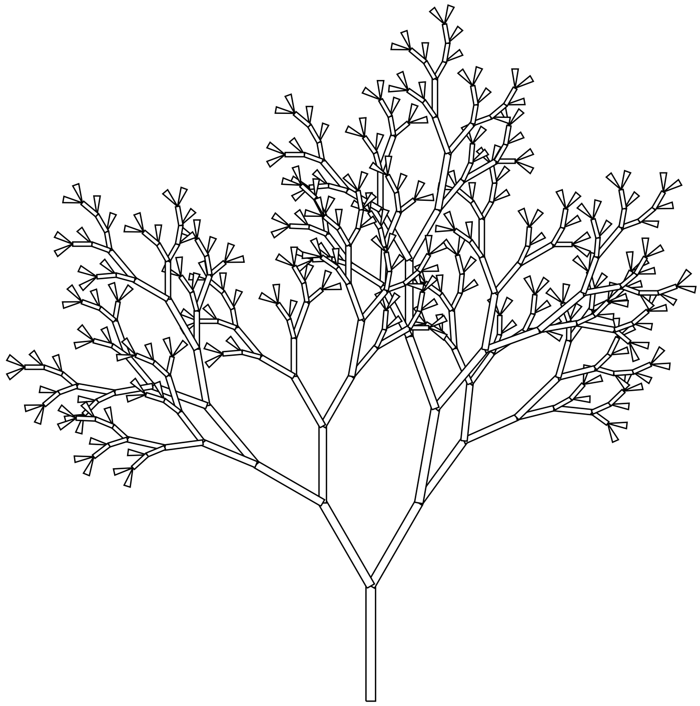
$e = 2.71828\,18285-$ (300 nodes)



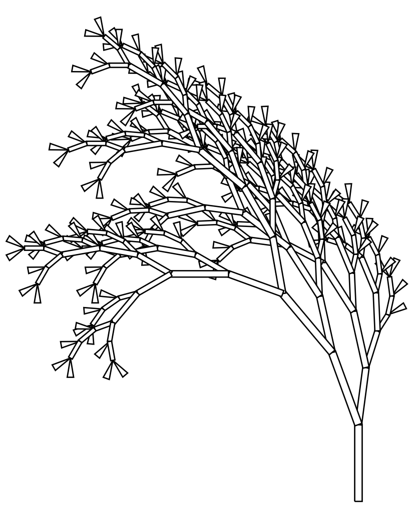
$1/e = 0.36787\,94412-$ (300 nodes)



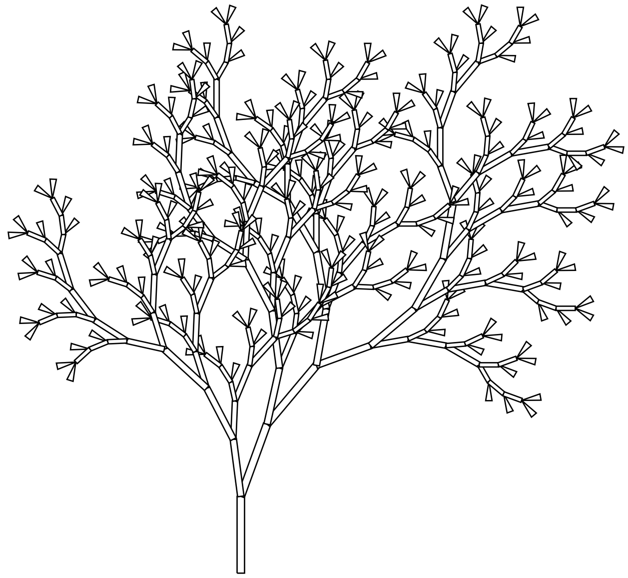
$\gamma = 0.57721\,56649+$ (300 nodes), possibly not irrational



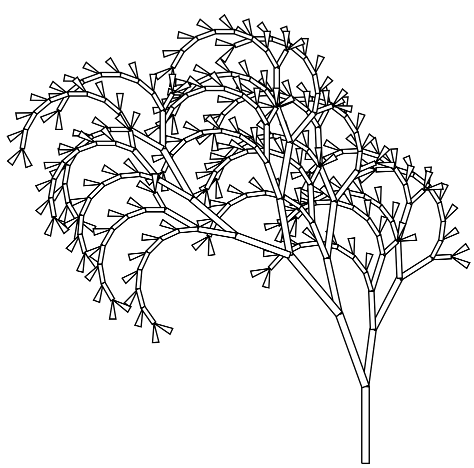
$\sqrt{2} = 1.41421\,35624-$ (250 nodes)



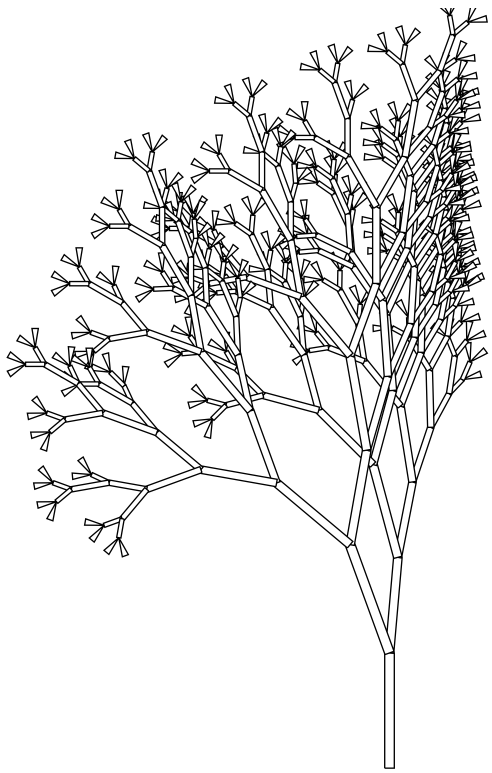
$\sqrt{3} = 1.73205\,08076-$ (200 nodes)



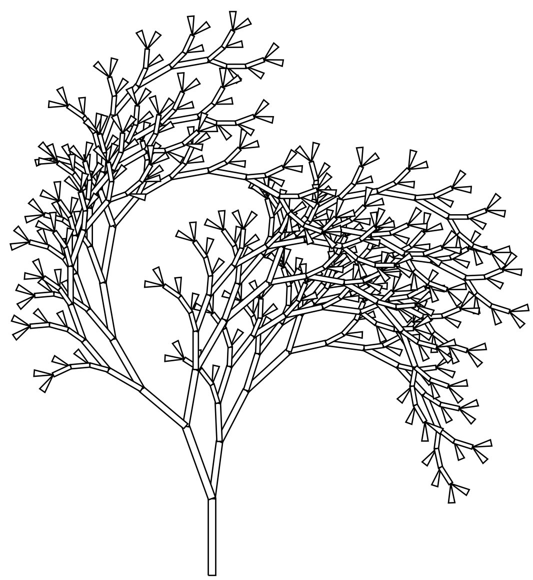
$\sqrt{5} = 2.23606\,79775-$ (250 nodes)



$\sqrt{\pi} = \Gamma(1/2) = 1.77245\,38509+$ (200 nodes)



$\phi = 1.61803\,59887+$ (300 nodes)



$\ln 2 = 0.69314\,71805+$ (250 nodes)